

PATENT

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

Serial No.: 09/849,691)
Filed: May 4, 2001)
For: Minimal Bends Connection Models)
For Wire Density Calculation)
Inventor: Alexander Tetelbaum)
Examiner: Mary C. Hogan)
Art Unit: 2123)
Atty. Ref.: 00-653)

DECLARATION OF INVENTOR UNDER 37 C.F.R. § 1.131

1. I am the inventor of the invention which is the subject of United States Patent Application Serial Number 09/849,691.
2. I conceived of the invention at least as early as March 2000.
3. I built a prototype of the invention at least as early as May 2000.
4. On or about August 1, 2000, I drafted an invention disclosure for my employer, LSI Logic. A copy of the invention disclosure is attached.
5. On or about October 10, 2000, I drafted an additional invention disclosure for my employer, LSI Logic. A copy of the invention disclosure is attached.

As a person signing below:

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code, and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

SIGNATURE

By: _____

Alexander Tetelbaum

Date: 1/17/05

I. Title of Your Invention: * A method of wire density calculation

Number of Attached Pages:

Was your Invention first presented in an LSI Logic "Brainstorming Session?" (Y) (unknown) (circle one)

If yes, specify the docket number assigned to you in the reminder e-mail: 00-000

II. List of All Inventors: (attach a separate sheet for additional inventors)

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Last Name: Telbaum

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Mail Stop: E-192

Work Phone: 67867

LSI Employee: YES

Vice President:

Thomas Daniel

Citizenship:
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If no, identify Employer:

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Hire Date: 3/9/1998

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Job Title: Project Leader, MTS

Department: 0000107424

Home Phone:
(510) 728-0265

ORIGINAL

III. Public Disclosure of Your Invention:

1. Was your Invention ever disclosed, either orally or in writing, to anyone other than an LSI Logic employee? (N)
2. If yes to 1, specify the date(s) of disclosure:
3. Are there plans to disclose your Invention in the future? (unknown)
4. If yes to 3, specify the date(s) of expected disclosure:

IV. Use of Your Invention:

1. Has your Invention been used? (Y) (N) (unknown)
2. If yes to 1, specify the date(s) of use*:
3. Are there plans to use your Invention in the future? (unknown)
4. If yes to 3, specify the date(s) of expected use:

V. Invention "offered for" or "on sale":

1. Was a product or process containing your Invention "offered for sale" or "sold?" (N)
2. If yes to 1, specify the date(s) of the offer or sale:

VI. Prior Art: (attach separate sheets if necessary) List only those patents, products, processes, journal articles, presentations, conferences, seminars, and other knowledge that you are aware of, that are related to the subject matter of your Invention: (you have no duty to conduct a search)

Author/Event/Product/Process:

Title:

Date:

1.
2.
3.
4.

VII. Background to Your Invention: (attach separate sheets with your responses)

Describe:

1. The field to which your Invention pertains.*
2. Problem(s) in the field which motivated your need to invent.*
3. Current approaches toward solving those problems (if any).*
4. Why those current approaches are unacceptable.*

VIII. Detailed Description of Your Invention: (attach separate sheets with your responses)

1. Provide enough information and detail so that another person in your field could make and use your Invention*.
 - If available, supplement your description with any existing reports, presentations, e-mails, sketches, drawings, schematics, photos, etc.
 - At least one simple Figure or Flowchart of your Invention MUST be included*.
2. Identify the new features of your Invention*.
3. List the advantages of your Invention*.
4. Disclose alternate ways of making and using of your Invention.

IX. Date of Your Invention:

1. Specify the date when you first conceived of your Invention*: (e.g. the conception date) Marth 2000
2. Specify the date the first prototype was built*: (e.g. reduction to practice) May 2000

X. Customer/Vendor Contracts:

1. Was your Invention developed during performance of a customer/vendor contract*? (N)

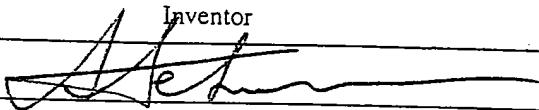
XI. Government March-In Rights:

1. Was your Invention conceived during performance of government contract*? (N)

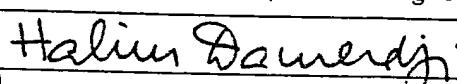
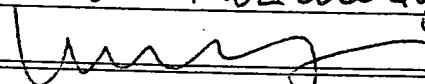
XII. Signatures*: (sign only in BLUE ink)

We the aforementioned inventors submit in confidence this Invention disclosure to Attorneys within the LSI Logic Legal Dept. for the purpose of obtaining a legal opinion and/or legal advice as to the availability of patent, trade secret, and/or copyright protection for, and/or a general legal opinion or legal advice relating to the material contained within.

I(We) believe myself(ourselves) to be the first and original inventor(s) of this Invention:

	Inventor	Date:
1. Alexander Tetelbaum		8/1/2000
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		

Two Witnesses who have read and understood this Invention disclosure*:

	Full Name of Witness (Print and Sign Name)	Date:
1. Halim Damerji		8/1/2000
2. Wei Huang		8/1/2000

The information within in this form has been provided to the LSI Logic Legal Department attorneys for the purpose of obtaining either a legal opinion, legal services, and/or assistance in a legal proceeding, and hence is privileged as an attorney-client communication.

VII. Background to Your Invention. (1. The field to which your Invention pertains., 2. Problem(s) in the field which motivated your need to invent., 3. Current approaches toward solving those problems (if any), 4. Why those current approaches are unacceptable.)

The invention is devoted to an optimal design of datapath macros with general and complex structures. (See invention disclosure 00-020). In a general case (see an example in Fig. 1) datapath cells are located in blocks (clusters) and control cells may be located in columns along left and right sides of the datapath and in blocks too.

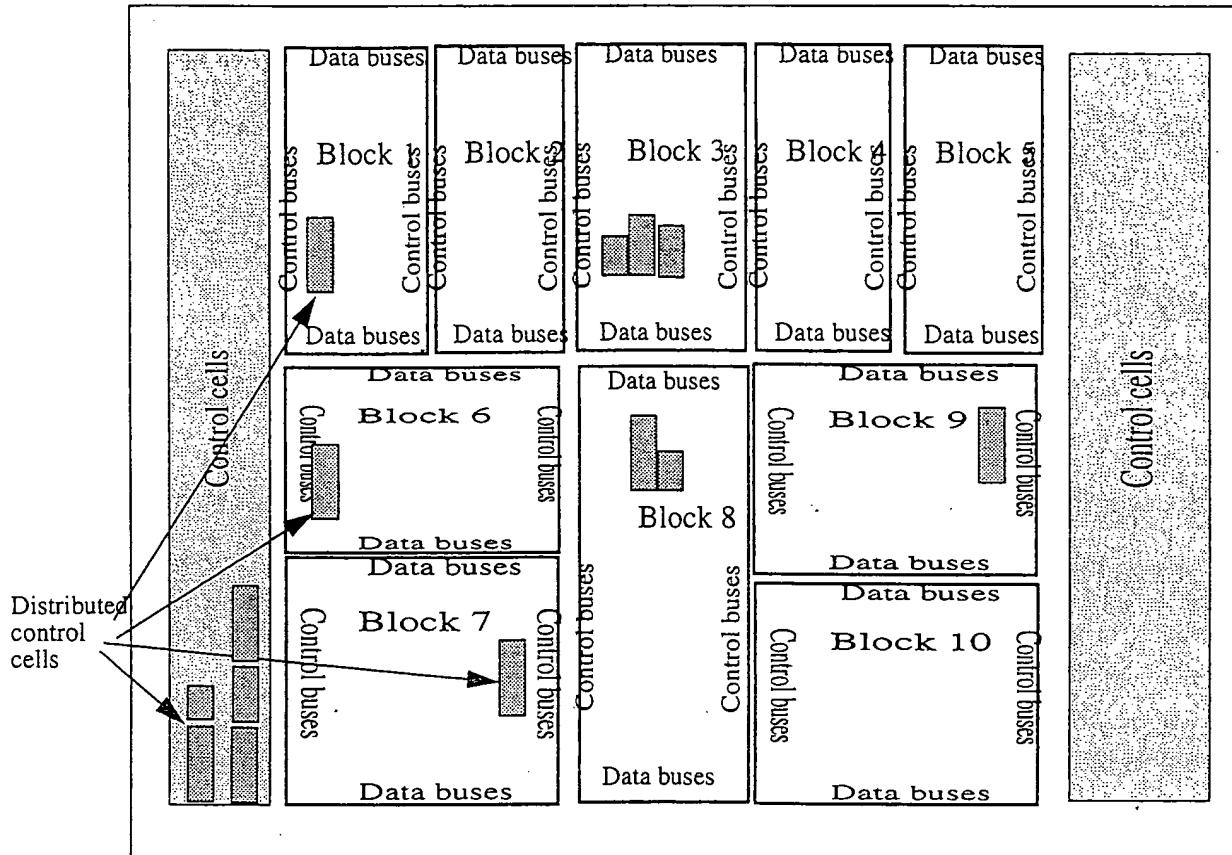


Figure 1. A general datapath structure.

Current datapaths are very complex and big designs that require:

- (1) Design datapath macros with complex hierarchical structures.
- (2) Respect complex input constraints on placement of cells, pins, nets, and gaps between cells, blocks, etc.
- (3) High quality of design of datapath macros.
- (4) Guarantee 100% detail routing.
- (5) Process very big and complex datapaths.

In particular, this invention is devoted to develop a probabilistic model for calculating wire density in different areas of the datapath. This model will be used in FS3.0 and future releases to form a congestion map and to introduce additional spaces between cells to meet the requirement 4 mentioned above.

The current approach (FS2.0) is based on a simplified density model and used for placement quality estimation only. This approach is not accurate and does not make a difference between vertical and horizontal segments of connections, and is unacceptable for calculating a congestion map.

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We suggest a new probabilistic method for calculating wire density in different areas of the datapath and other hardmacs with a given cell placement. This method is based on a new probabilistic model of connection between two pins. This model takes into account all possible shortest length configurations of the connection and makes a difference between vertical and horizontal segments of the connection. Thus, this model is accurate enough to be used for wire density estimation.

We divide the datapath into M_{DP} by N_{DP} squared areas as it is shown in Fig. 2, where each area size is about equal to the width of placement columns (or cell width).

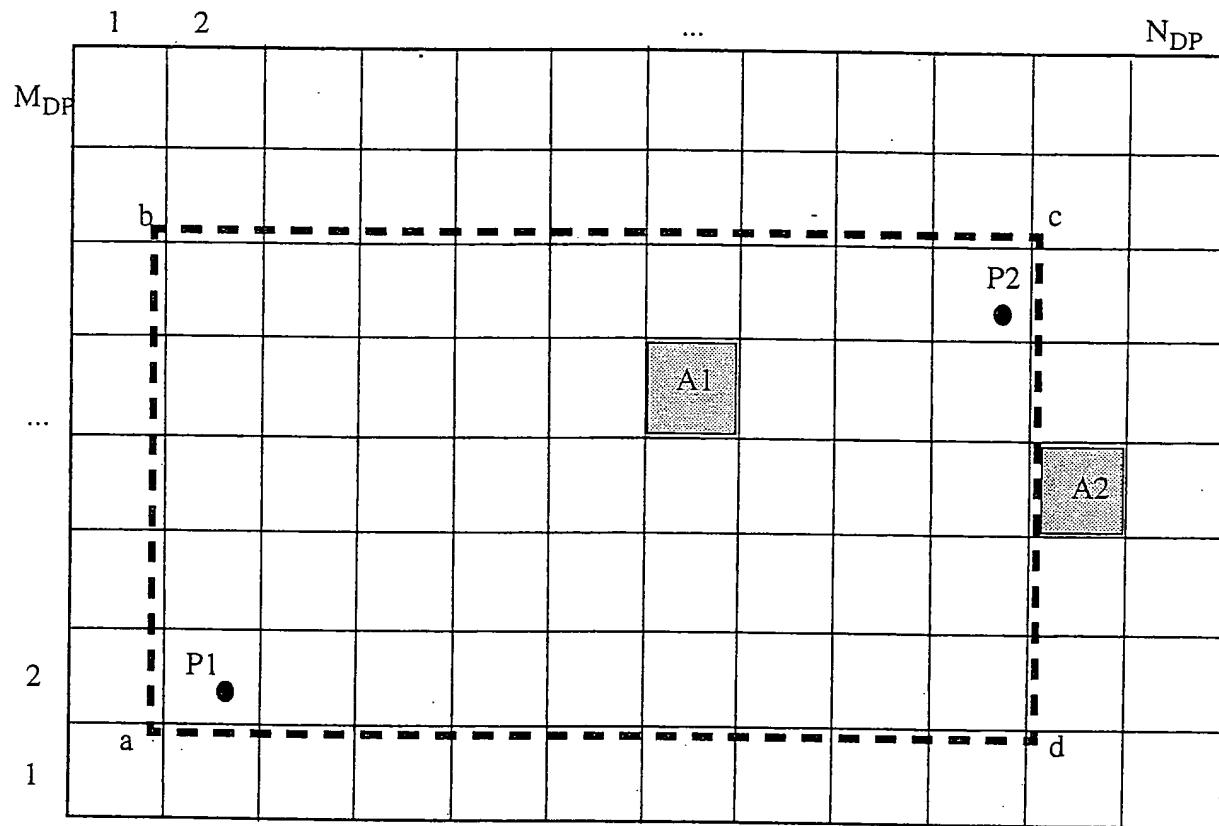


Figure 2. Datapath areas.

It is known that the number of the shortest length paths (configurations) from P1 to P2 (see Fig. 3) is

$$N(P1, P2) = \binom{m-1}{n+m-2} = \frac{(m+n-2)!}{(m-1)! \cdot (n-1)!}$$

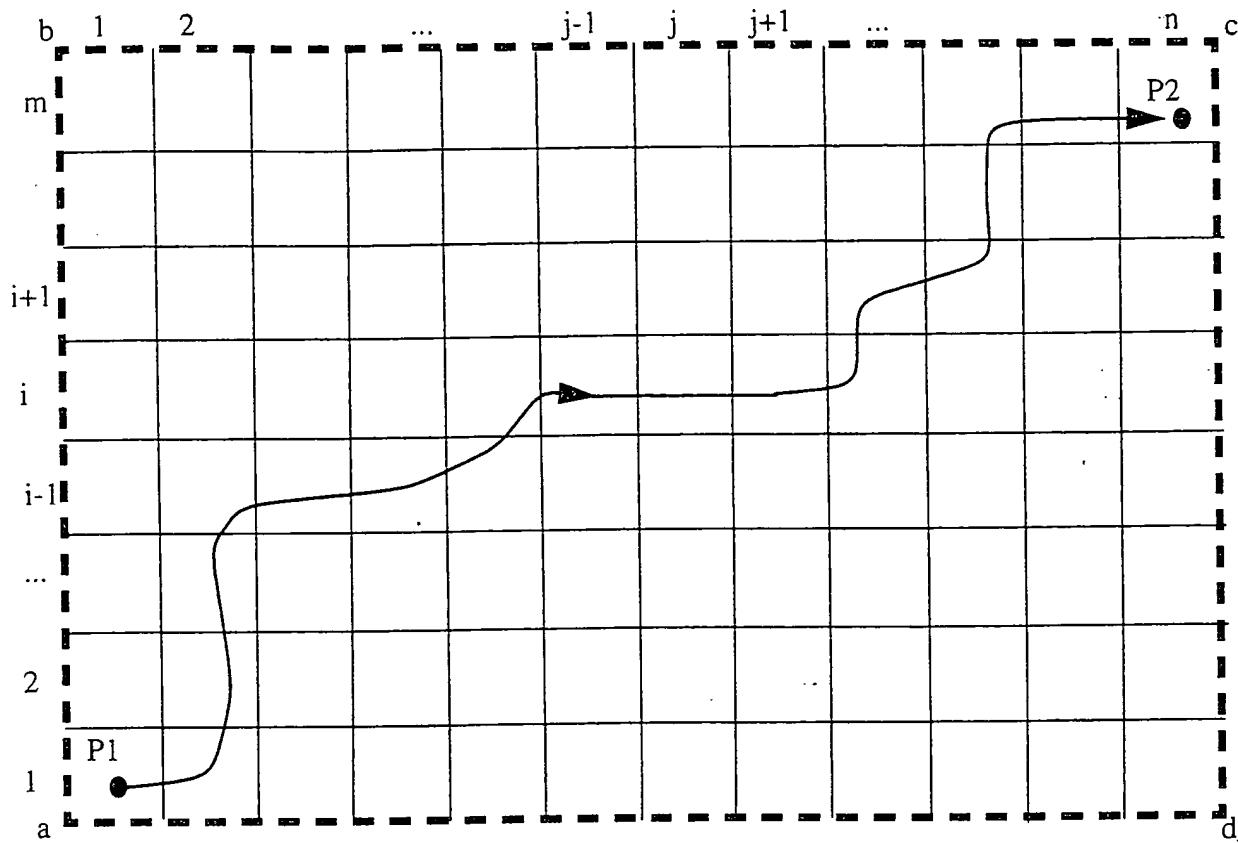


Figure 3. Rectangle [a,b,c,d].

We start with studying possible shortest length configurations for connection C from pin P1 to pin P2 (see Fig. 1). It is known that any shortest length connection (P1,P2) will belong to rectangle [a,b,c,d]. Therefore, for any area A1 from [a,b,c,d] there exists some probability $P(A1) > 0$ that connection (P1,P2) will go thru this area, and for any area A2 outside [a,b,c,d] there is zero probability that connection (P1,P2) will go thru this area. If we know probability $P(A)$ then the mathematical expectation of area A having connection (P1,P2) is $M(A) = P(A)$.

Any connection (P1,P2) can go thru area A in 6 different ways as it is shown in Fig. 4. The last configurations (e and f) are possible if pin P1 is higher than pin P2.

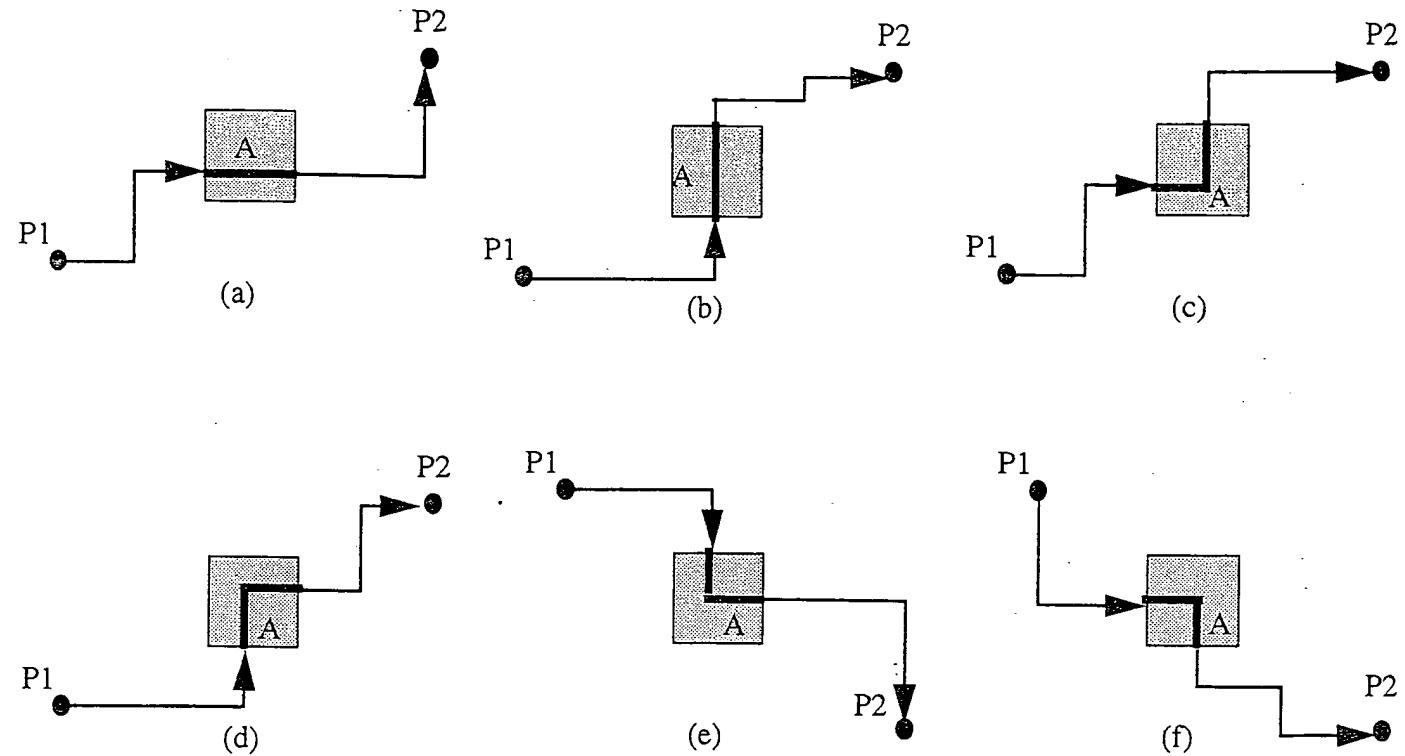


Figure 4. Possible connection configurations in area A.

we will consider pins P1 and P2 placed as it is shown in Fig.2. For the location when P1 is higher than P2 everything will be analogous. Let us find mathematical expectations of horizontal $M_h(A)$ and vertical $M_v(A)$ segments for connection (P1,P2) in area A. Let us start with the horizontal segments. Fig. 5 shows the rectangle [a,b,c,d] (see Fig. 2) with the full horizontal segment (see Fig. 4a) and the numeration of its columns and rows. The mathematical expectations $M_{h1}(A)$ of full horizontal segments can be calculated as follows

$$M_{h1}(A) = \frac{N(P1, A') \cdot N(A'', P2)}{N(P1, P2)}$$

where

$N(P1, A')$ is the number of possible paths from P1 to area A'

$N(A'', P2)$ is the number of possible paths from area A'' to P2

$N(P1, P2)$ is the number of possible paths from P1 to P2.

I. Title of Your Invention: * Minimal bends connection models for wire density calculation

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If yes, specify the docket number assigned to you in the reminder e-mail: 00-025

II. List of All Inventors: (attach a separate sheet for additional inventors)

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Work Phone: 67867

LSI Employee: YES

Vice President:
Thomas Daniel

Citizenship:
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Work Fax: 64156

If no, identify Employer:

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Department: 0000107767

Hire Date: 3/9/1998

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Job Title: Project Leader, MTS

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LSI LOGIC CORP.
INTELLECTUAL PROPERTY DEPT.

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LSI Employee: YES

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Citizenship:
USA

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Job Title: Project Leader, MTS

Department: 0000107424

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(510) 728-0265

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Date:

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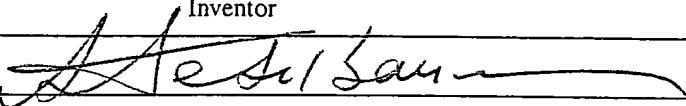
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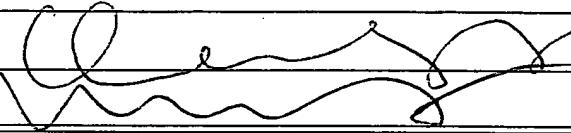
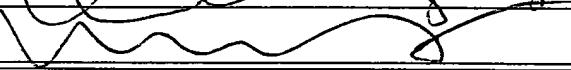
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I(We) believe myself(ourselves) to be the first and original inventor(s) of this Invention:

	Inventor	Date:
1. Alexander Tetelbaum		10/10/2000
2.		
3.		
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10.		

Two Witnesses who have read and understood this Invention disclosure*:

	Full Name of Witness (Print and Sign Name)	Date:
1. John Yu		10/10/2000
2. Wei Huang		10/10/2000

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VII. Background to Your Invention: (1. The field to which your Invention pertains., 2. Problem(s) in the field which motivated your need to invent., 3. Current approaches toward solving those problems (if any)., 4. Why those current approaches are unacceptable.)

The invention is devoted to an optimal design of complex chips. In particular, this invention is devoted to development of probabilistic models for calculating wire density in different areas of the chip after cell placement. This model will be used in FS3.0 and future releases to form a congestion map and to introduce additional spaces between cells to guarantee 100% detail routing.

The current approach (FS2.0) is based on a simplified density model and is used for placement quality estimation only. This approach is not accurate and does not make a difference between vertical and horizontal segments of connections, and is unacceptable for calculating a congestion map. A new approach was developed for FS3.0 and described in invention disclosure 00-448. This approach is a good probabilistic model for connections going thru areas with high wire density. This model takes into account all possible shortest length configurations of the connection and makes a difference between vertical and horizontal segments of the connection. This approach has two following drawbacks for chip areas with low and middle wire density. First, it assumes that the connection can have any configuration with the same probability. This is not always true: the connection is more likely has a configuration with small number of bends in chip areas with low and middle wire density. Second, the probability of any connection configuration that goes thru or near the center of the bounding box (see Fig. 1) around the connection is higher that for other configurations. This is also not always true: it will depend on location of other pins and wires.

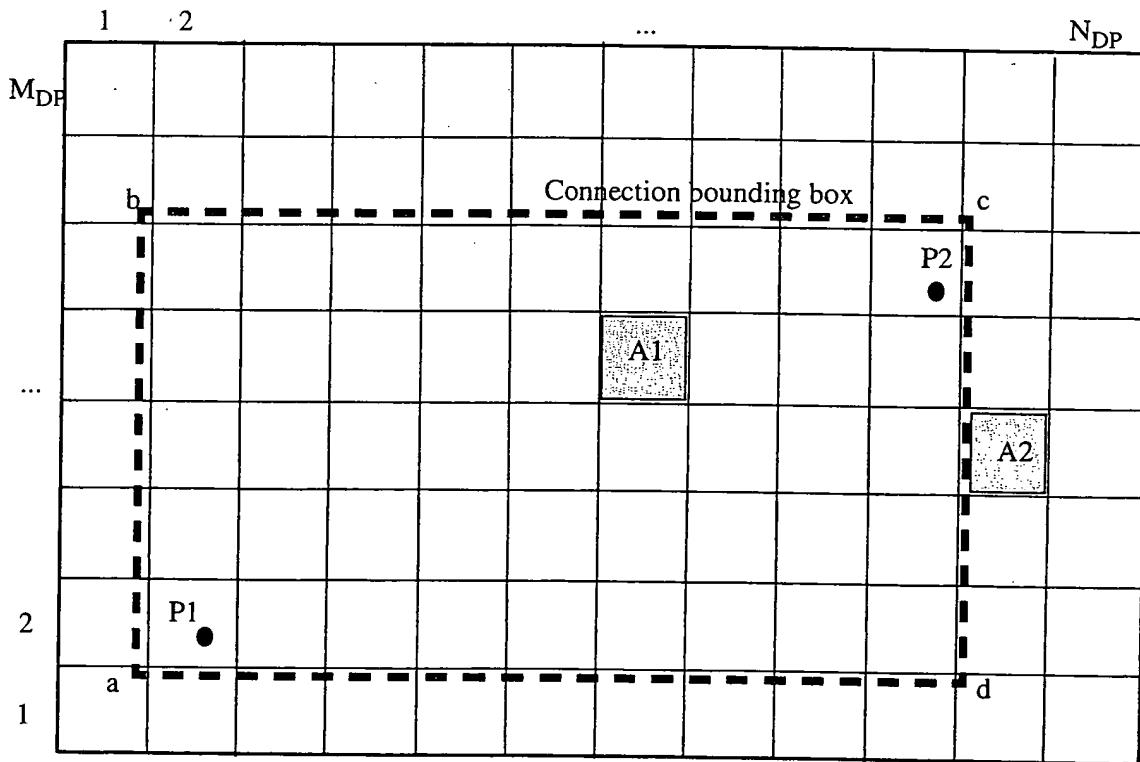


Figure 1. Chip areas.

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We suggest a new method for calculating wire density in different areas of chips or hardmacs with a given cell placement. This method is based on a set of new probabilistic models of connection between two pins. These models take into account all possible minimal bends and shortest length configurations of the connection and make a difference between vertical and horizontal segments of the connection. Thus, these models are accurate enough to be used for wire density estimation in areas with low, middle, and high wire density. The method suggests to use model with the minimum bends in areas with low wire density, and models with more bends in areas with middle and high wire density. The rule is “the more wire density the more bends in the model”. We first find the model with the minimum bends and then recursively use this model build other model with more bends.

We divide the chip into M_{DP} by N_{DP} squared areas as it is shown in Fig. 1, where each area size is about equal to the width of placement columns (or cell width). Note, that in Avant! tool the chip is rotated 90 degree and we have not placement columns, but rather placement rows.

We start with studying the minimum bends model (model 1) that describes all connection configurations with only one bend and the shortest length. See these configurations for connection C from pin P1 to pin P2 in Fig. 2.

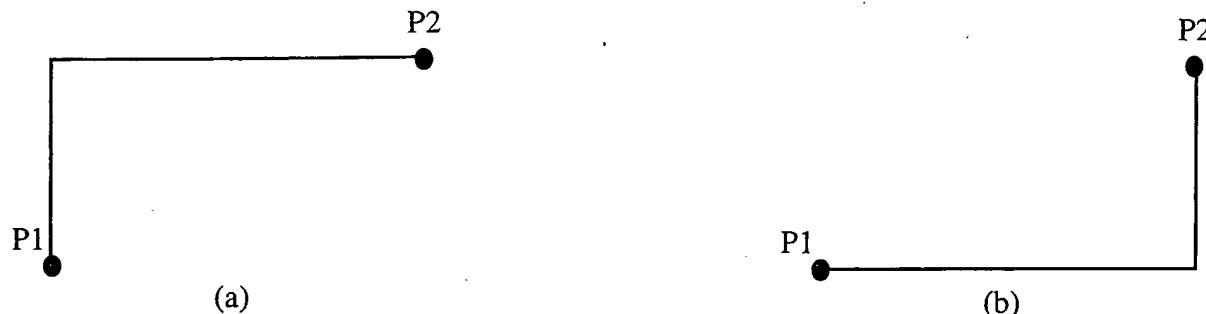


Figure 2. Possible minimum bends configurations.

Let us find the probability $P(A)$ for each area A of the connection bounding box $[a,b,c,d]$ to have the connection (P_1, P_2) . See Fig. 3. For any area A_1 from $[a,b,c,d]$ there exists some probability $P(A) \geq 0$ that connection (P_1, P_2) will go thru this area, and for any area A outside $[a,b,c,d]$ there is zero probability that connection (P_1, P_2) will go thru this area. If we know probability $P(A)$ then the mathematical expectation of area A having connection (P_1, P_2) is $M(A) = P(A)$.

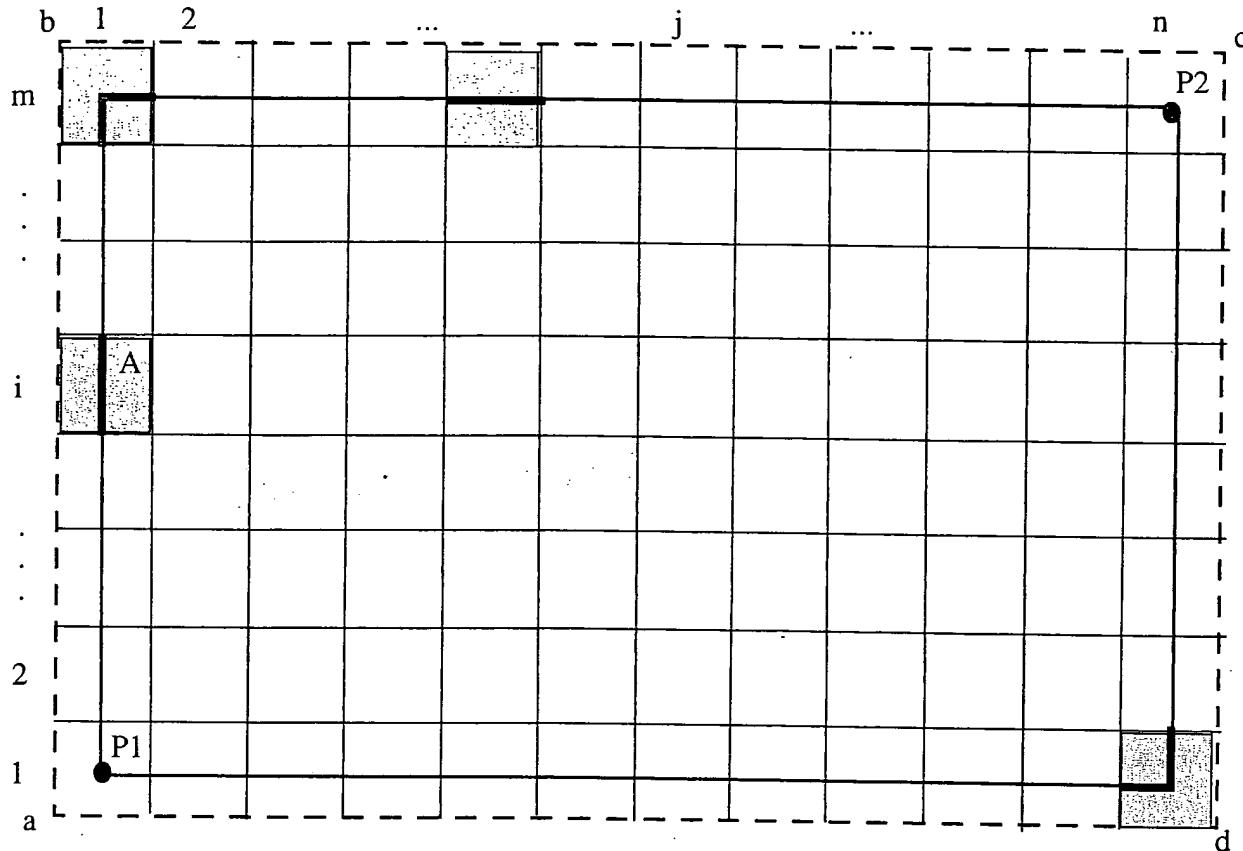


Figure 3. Area configurations.

Any connection (P1,P2), where P1 is lower than P2, can go thru area A in 3 different ways as it is shown in Fig. 3. We will consider pins P1 and P2 placed as it is shown in Fig.2. For the location when P1 is higher than P2 everything will be analogous. Let us find mathematical expectations of horizontal $M_h(A)$ and vertical $M_v(A)$ segments for connection (P1,P2) in different areas A. Let us start with the horizontal segments. Fig. 4a shows the bounding box $[a,b,c,d]$ (see Fig. 3) with the enumeration of its columns and rows, and all horizontal probabilities (mathematical expectations) for the configuration in Fig. 2a. Areas with a full horizontal segment have 0.5 probability, because there are only two possible configurations. Areas with a half horizontal segment have 0.25 probability, because these areas contain about 0.5 part of the segment and there are only two possible configurations. Fig. 4a shows all horizontal probabilities for configuration in Fig. 2b.

b	1	2	...	j	...	n	c	
m	0.25	0.5	0.5	...	0.5	...	0.5	0.5
1	0	0	...	0	...	0	0	0
i
2	0	0	...	0	...	0	0	0
1	P_1	0	0	...	0	...	0	0
a
d

(a)

b	1	2	...	j	...	n	c
m	0	0	...	0	...	0	0
1	0	0	...	0	...	0	0
i
2	0	0	...	0	...	0	0
1	P_1	0.25	0.5	0.5	...	0.5	0.5
a
d

(b)

Figure 4. Horizontal probabilities.

The whole mathematical expectation $M_h(A)$ can be found as a sum

$$M_h(A) = M_{h1}(A) + M_{h2}(A)$$

of mathematical expectations for both configurations in Fig. 4. See Fig. 5. The formula for mathematical expectation $M_h(A)$ is as follows

$$M_h(A) = 0.5 \text{ if } i = 1 \text{ and } j = 2, 3, \dots, n-1$$

$$M_h(A) = 0.5 \text{ if } i = m \text{ and } j = 2, 3, \dots, n-1$$

$$M_h(A) = 0.25 \text{ if } i = 1 \text{ and } j = 1 \text{ or } j = n$$

$$M_h(A) = 0 \quad \text{if } i = 2, 3, \dots, m-1 \text{ and } j = 1, 2, \dots, n,$$

where we use local (inside [abcd]) numeration of rows and columns.

b	1	2	...	j	...	n	c	
m	0.25	0.5	0.5	...	0.5	...	0.5	0.25
i	0	0	...	0	...	0	0	
j	0.25	0.5	0.5	...	0.5	...	0.5	0.25
d	P1	0.25	0.5	0.5	...	0.5	0.5	0.25
a	0.25	0.5	0.5	...	0.5	...	0.5	0.25

Figure 5. Horizontal mathematical expectations.

The same formulas can be used for horizontal segments when point P1 is higher than point P2.

To find mathematical expectation $M_h^{All}(A)$ of all horizontal segments of the all connections we do the summation

$$M_h^{All}(A) = \sum_{c \in \text{Connections}} M_h^c(A)$$

where $M_h^c(A) = M_h(A)$ is the whole mathematical expectation of horizontal segments in area A for one connection c.

The same approach can be used to obtain formulas for vertical segments (see Fig. 6):

$$M_v(A) = 0.5 \text{ if } j = 1 \text{ and } i = 2, 3, \dots, m-1$$

$$M_v(A) = 0.5 \text{ if } j = m \text{ and } i = 2, 3, \dots, m-1$$

$$M_v(A) = 0.25 \text{ if } j = 1 \text{ and } i = 1 \text{ or } i = m$$

$$M_v(A) = 0 \text{ if } j = 2, 3, \dots, n-1 \text{ and } i = 1, 2, \dots, m,$$

where we use local (inside [abcd]) numeration of rows and columns.

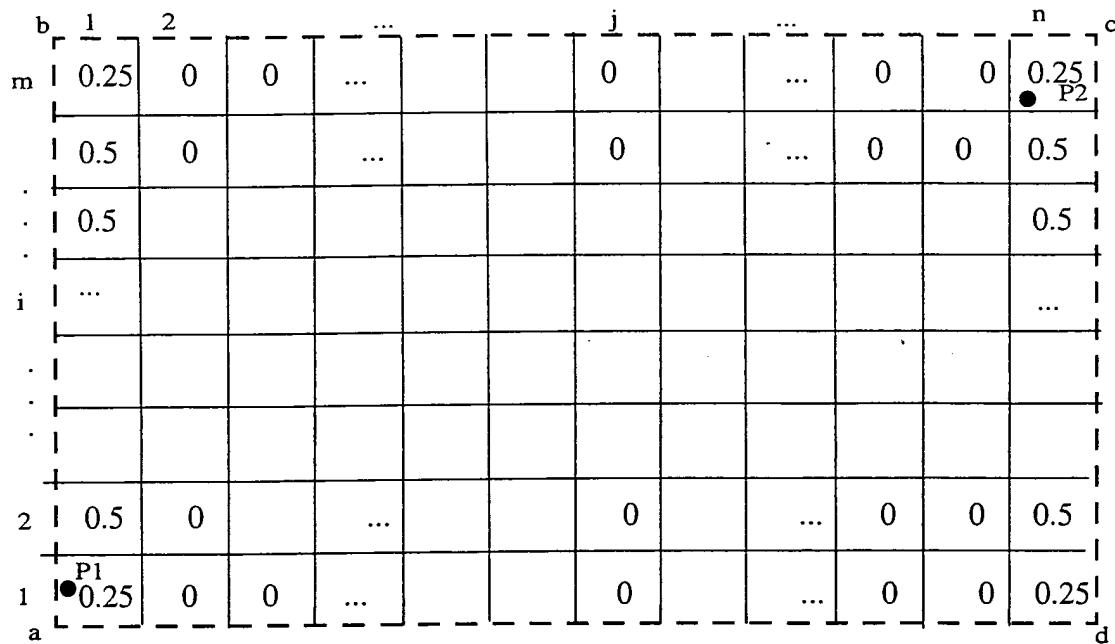


Figure 6. Vertical mathematical expectations.

The same formulas can be used for vertical segments when point P1 is higher than point P2.

To find mathematical expectation $M_v^{\text{All}}(A)$ of all vertical segments of the all connections we do the summation

$$M_v^{\text{All}}(A) = \sum_{c \in \text{Connections}} M_v^c(A)$$

From the formulas above we can conclude that the time complexity of the model will depend on n and m. The time complexity for one connection is $O(m+n)$. The time complexity for all N connections is $O(N(m+n))$.

Now, we want to recursively use the obtained formulas for one bend configuration to find models with 2, 3,... bends. Let us consider a connection configuration with two bends. There are 2 possible types of these configuration. See Fig. 7.

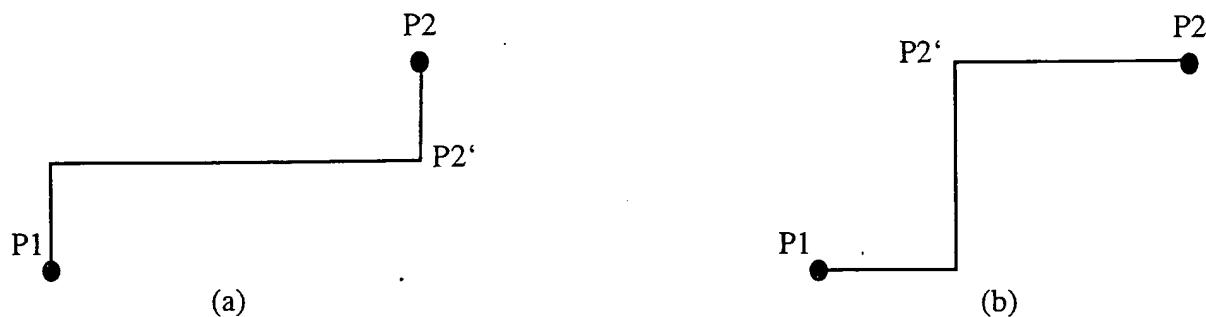


Figure 7. Possible 2-bends configurations.

To find all mathematical expectations we can consider 2-bends configuration as a combination of all possible 1-bend configurations ($P1, P2'$). It is seen that there are m possible locations for $P2'$ for configurations in Fig. 7a and there are n possible locations for $P2'$ for configurations in Fig. 7b. The whole mathematical expectation $M_h(A)$ can be found as a sum

$$M_h(A) = M_{h1}(A) + M_{h2}(A) + \dots + M_{h(m+n)}(A)$$

of mathematical expectations for all possible configurations in Fig. 7. Fig. 8 shows all possible configurations in Fig. 7a.

b	1	2	...	j	...	n	c
m	0.25/m	0.5/m	0.5/m	...	0.5/m	...	0.25/m
i	0.25/m	0.5/m	0.5/m	...	0.5/m	...	0.25/m
2	0.25/m	0.5/m	0.5/m	...	0.5/m	...	0.25/m
1	0.25/m	0.5/m	0.5/m	...	0.5/m	...	0.25/m
a	0.25/m	0.5/m	0.5/m	...	0.5/m	...	0.25/m

Figure 8. Horizontal mathematical expectations for Fig. 7a configurations.

Fig. 9 shows all possible configurations in Fig. 7b.

b	1	2	...	j	...	n	c
m	0.25 n	1 n	1.5 n	...	0.5j n	...	0.25 n
i	0	0	...	0	...	0	0
2	0	0	...	0	...	0	0
1	0.25 n	0.5(n-1) n	0.5(n-2) n	...	0.5(n-j+1) n	...	1.5 n
a	P1						0.25 n

Figure 9. Horizontal mathematical expectations for Fig. 7b configurations.

Fig. 10 shows the whole mathematical expectations for both configurations in Fig. 7.

b	1 $\frac{0.25(m+n)}{nm}$	2 $\frac{0.5(2m+n)}{nm}$	3 $\frac{0.5(3m+n)}{nm}$...	$\frac{0.5(nm+n-2m)}{nm}$	$\frac{0.5(nm+n-m)}{nm}$	$\frac{0.25(m+1)}{nm}$	c
m	0.25/m	0.5/m	0.5/m	...	0.5/m	0.5/m	0.25/m	P2
i	...							
	0.25/m	0.5/m	0.5/m	...	0.5/m	0.5/m	0.25/m	
2	0.25/m	0.5/m	0.5/m	...	0.5/m	0.5/m	0.25/m	
1	$\frac{0.25(m+1)}{nm}$	$\frac{0.5(mn+n-m)}{nm}$	$\frac{0.5(nm+n-2m)}{nm}$...	$\frac{0.5(3m+n)}{nm}$	$\frac{0.5(2m+n)}{nm}$	$\frac{0.25(m+n)}{nm}$	
a	P1							d

Figure 10. Horizontal mathematical expectations.

The formula for mathematical expectation $M_h(A)$ is as follows

$$M_h(A) = 0.5(mn+n-(j-1)m)/nm \quad \text{if } i = 1 \text{ and } j = 2, 3, \dots, n-1$$

$$M_h(A) = 0.5(jm+n)/nm \quad \text{if } i = m \text{ and } j = 2, 3, \dots, n-1$$

$$M_h(A) = 0.25(m+1)/m \quad \text{if } i = 1 \text{ and } j = 1$$

$$M_h(A) = 0.25(n+m)/nm \quad \text{if } i = m \text{ and } j = 1 \text{ or } j = n$$

$$M_h(A) = 0.25(m+1)/nm \quad \text{if } i = m \text{ and } j = n$$

$$M_h(A) = 0.5/m \quad \text{if } i = 2, 3, \dots, m-1 \text{ and } j = 1, 2, \dots, n,$$

where we use local (inside [abcd]) numeration of rows and columns.

The same formulas can be used for horizontal segments when point P1 is higher than point P2.

The same approach can be used to obtain formulas for vertical segments:

$$M_v(A) = 0.5(mn+m-(i-1)n)/nm \quad \text{if } j = 1 \text{ and } i = 2, 3, \dots, m-1$$

$$M_v(A) = 0.5(in+m)/nm \quad \text{if } j = n \text{ and } i = 2, 3, \dots, m-1$$

$$M_v(A) = 0.25(n+1)/n \quad \text{if } j = 1 \text{ and } i = 1$$

$$M_v(A) = 0.25(n+m)/nm \quad \text{if } j = n \text{ and } i = 1 \text{ or } i = m$$

$$M_v(A) = 0.25(n+1)/nm \quad \text{if } j = n \text{ and } i = m$$

$$M_v(A) = 0.5/n \quad \text{if } j = 2, 3, \dots, n-1 \text{ and } i = 1, 2, \dots, m,$$

where we use local (inside [abcd]) numeration of rows and columns.

From the formulas above we can conclude that the time complexity of the model will depend on n and m. The time complexity for one connection and one area is $O(mn)$. The time complexity for all N connections is $O(Nmn)$. We see that with the increase of bends in the model, the time complexity increases too.

Finally, let us outline the 3-bends model. Again, we will use the obtained formulas for 2-bend configuration to find the model with 3 bends. There are 2 possible types of these configuration. See Fig. 11.



Figure 11. Possible 3-bends configurations.

To find all mathematical expectations we can consider 3-bends configuration as a combination of all possible 2-bends configurations ($P1, P2'$). It is seen that there are n possible locations for $P2'$ for configurations in Fig. 11a and there are m possible locations for $P2'$ for configurations in Fig. 11b.

Using this approach recursively we can theoretically build a model with any given number of bends. Practically, the calculations to build k -bends ($k \geq 3$) models may become too expensive and an improvement in accuracy may be small. To speed up all calculations we can tabulate all possible matrices for the mathematical expectations for given sizes of connections m and n . There will be mn matrices for each model. Then, for any k -bends model the time complexity for all N connections is always $O(Nmn)$. Note, that we ignore the time for tabulation, because we do it only once.

The main value of the invention is that these time efficient models can be used for an accurate estimation of horizontal and vertical wire density in different areas of datapath or hardmac. These models take into account all possible minimal bends and shortest length configurations of the connection. Thus, these models are accurate enough to be used for wire density estimation in areas with low, middle, and high wire density. The method suggests to use model with the minimum bends in areas with low wire density, and models with more bends in areas with middle and high wire density.